[2 次元一般座標における流れの拡散項について]

・X方向

$$\frac{\partial}{\partial x}\left(\nu_T \frac{\partial u^x}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_T \frac{\partial u^x}{\partial y}\right)$$

・Y方向

$$\frac{\partial}{\partial x}\left(\nu_T \frac{\partial u^y}{\partial x}\right)+\frac{\partial}{\partial y}\left(\nu_T \frac{\partial u^y}{\partial y}\right)$$

(ξ,η) を一般座標とする

$$f_x=\xi_x\frac{\partial f}{\partial \xi}+\eta_x\frac{\partial f}{\partial \eta}\quad ,\qquad f_y=\xi_y\frac{\partial f}{\partial \xi}+\eta_y\frac{\partial f}{\partial \eta}$$

—— 式を変換すると

・X方向

$$\left(\xi_x\frac{\partial}{\partial \xi}+\eta_x\frac{\partial}{\partial \eta}\right)\left(\nu_T\left(\xi_x\frac{\partial u^x}{\partial \xi}+\eta_x\frac{\partial u^x}{\partial \eta}\right)\right)+\left(\xi_y\frac{\partial}{\partial \xi}+\eta_y\frac{\partial}{\partial \eta}\right)\left(\nu_T\left(\xi_y\frac{\partial u^x}{\partial \xi}+\eta_y\frac{\partial u^x}{\partial \eta}\right)\right)$$

・Y方向

$$\left(\xi_x\frac{\partial}{\partial \xi}+\eta_x\frac{\partial}{\partial \eta}\right)\left(\nu_T\left(\xi_x\frac{\partial u^y}{\partial \xi}+\eta_x\frac{\partial u^y}{\partial \eta}\right)\right)+\left(\xi_y\frac{\partial}{\partial \xi}+\eta_y\frac{\partial}{\partial \eta}\right)\left(\nu_T\left(\xi_y\frac{\partial u^y}{\partial \xi}+\eta_y\frac{\partial u^y}{\partial \eta}\right)\right)$$

—— $\xi_{x\xi}$, $\xi_{y\xi}$, $\xi_{x\eta}$, $\xi_{y\eta}$, $\eta_{x\xi}$, $\eta_{y\xi}$, $\eta_{x\eta}$, $\eta_{y\eta}$ ＝ 0 と仮定すると

・ξ方向 [X方向×ξ_x ＋Y方向×ξ_y]

$$\left(\xi_x\frac{\partial}{\partial \xi}+\eta_x\frac{\partial}{\partial \eta}\right)\left(\nu_T\left(\xi_x\frac{\partial u^x\xi_x+u^y\xi_y}{\partial \xi}+\eta_x\frac{\partial u^x\xi_x+u^y\xi_y}{\partial \eta}\right)\right)$$
$$+\left(\xi_y\frac{\partial}{\partial \xi}+\eta_y\frac{\partial}{\partial \eta}\right)\left(\nu_T\left(\xi_y\frac{\partial u^x\xi_x+u^y\xi_y}{\partial \xi}+\eta_y\frac{\partial u^x\xi_x+u^y\xi_y}{\partial \eta}\right)\right)$$

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・η方向　[X方向×η_x＋Y方向×η_y]

$$\left(\xi_x\frac{\partial}{\partial\xi}+\eta_x\frac{\partial}{\partial\eta}\right)\left(\nu_T\left(\xi_x\frac{\partial u^x\eta_x+u^y\eta_y}{\partial\xi}+\eta_x\frac{\partial u^x\eta_x+u^y\eta_y}{\partial\eta}\right)\right)$$
$$+\left(\xi_y\frac{\partial}{\partial\xi}+\eta_y\frac{\partial}{\partial\eta}\right)\left(\nu_T\left(\xi_y\frac{\partial u^x\eta_x+u^y\eta_y}{\partial\xi}+\eta_y\frac{\partial u^x\eta_x+u^y\eta_y}{\partial\eta}\right)\right)$$

――　ここで　$u^\xi=\left(u^x\xi_x+u^y\xi_y\right)$，　$u^\eta=\left(u^x\eta_x+u^y\eta_y\right)$　と定義すると

・ξ方向

$$\left(\xi_x\frac{\partial}{\partial\xi}+\eta_x\frac{\partial}{\partial\eta}\right)\left(\nu_T\left(\xi_x\frac{\partial u^\xi}{\partial\xi}+\eta_x\frac{\partial u^\xi}{\partial\eta}\right)\right)+\left(\xi_y\frac{\partial}{\partial\xi}+\eta_y\frac{\partial}{\partial\eta}\right)\left(\nu_T\left(\xi_y\frac{\partial u^\xi}{\partial\xi}+\eta_y\frac{\partial u^\xi}{\partial\eta}\right)\right)$$

・η方向

$$\left(\xi_x\frac{\partial}{\partial\xi}+\eta_x\frac{\partial}{\partial\eta}\right)\left(\nu_T\left(\xi_x\frac{\partial u^\eta}{\partial\xi}+\eta_x\frac{\partial u^\eta}{\partial\eta}\right)\right)+\left(\xi_y\frac{\partial}{\partial\xi}+\eta_y\frac{\partial}{\partial\eta}\right)\left(\nu_T\left(\xi_y\frac{\partial u^\eta}{\partial\xi}+\eta_y\frac{\partial u^\eta}{\partial\eta}\right)\right)$$

――　$\xi_{x\xi}$，$\xi_{y\xi}$，$\xi_{x\eta}$，$\xi_{y\eta}$，$\eta_{x\xi}$，$\eta_{y\xi}$，$\eta_{x\eta}$，$\eta_{y\eta}$　=　0　の仮定から

ξ方向の一部を展開すると

$$\xi_x\frac{\partial}{\partial\xi}\left(\nu_T\xi_x\frac{\partial u^\xi}{\partial\xi}\right)=\frac{\partial}{\partial\xi}\left(\nu_T\xi_x{}^2\frac{\partial u^\xi}{\partial\xi}\right)$$

$$\eta_x\frac{\partial}{\partial\eta}\left(\nu_T\xi_x\frac{\partial u^\xi}{\partial\xi}\right)=\frac{\partial}{\partial\eta}\left(\nu_T\eta_x\xi_x\frac{\partial u^\xi}{\partial\xi}\right)$$

となるので、同様にすべてのξ方向の式を展開すると、

[2 次元一般座標における流れの拡散項について]

$$\frac{\partial}{\partial \xi}\left\{\nu_T \xi_x{}^2 \frac{\partial u^\xi}{\partial \xi}\right\}+\frac{\partial}{\partial \xi}\left\{\nu_T \xi_y{}^2 \frac{\partial u^\xi}{\partial \xi}\right\}+\frac{\partial}{\partial \eta}\left\{\nu_T \xi_x \eta_x \frac{\partial u^\xi}{\partial \xi}\right\}+\frac{\partial}{\partial \eta}\left\{\nu_T \xi_y \eta_y \frac{\partial u^\xi}{\partial \xi}\right\}$$

$$+\frac{\partial}{\partial \xi}\left\{\nu_T \xi_x \eta_x \frac{\partial u^\xi}{\partial \eta}\right\}+\frac{\partial}{\partial \xi}\left\{\nu_T \xi_y \eta_y \frac{\partial u^\xi}{\partial \eta}\right\}+\frac{\partial}{\partial \eta}\left\{\nu_T \eta_x{}^2 \frac{\partial u^\xi}{\partial \eta}\right\}+\frac{\partial}{\partial \eta}\left\{\nu_T \eta_y{}^2 \frac{\partial u^\xi}{\partial \eta}\right\}$$

――　上式を整理すると、

$$\frac{\partial}{\partial \xi}\left\{\nu_T\left(\xi_x{}^2+\xi_y{}^2\right)\frac{\partial u^\xi}{\partial \xi}\right\}+\frac{\partial}{\partial \eta}\left\{\nu_T\left(\xi_x \eta_x+\xi_y \eta_y\right)\frac{\partial u^\xi}{\partial \xi}\right\}$$

$$+\frac{\partial}{\partial \xi}\left\{\nu_T\left(\xi_x \eta_x+\xi_y \eta_y\right)\frac{\partial u^\xi}{\partial \eta}\right\}+\frac{\partial}{\partial \eta}\left\{\nu_T\left(\eta_x{}^2+\eta_y{}^2\right)\frac{\partial u^\xi}{\partial \eta}\right\}$$

ここでξ軸とη軸が直交していると仮定すると

$$\xi_x{}^2+\xi_y{}^2=\xi_r{}^2\tilde{\xi}_x{}^2+\xi_r{}^2\tilde{\xi}_y{}^2=\xi_r{}^2\left(\tilde{\xi}_x{}^2+\tilde{\xi}_y{}^2\right)=\xi_r{}^2\left(\cos^2\theta+\sin^2\theta\right)=\xi_r{}^2 \quad ,$$

$$\xi_x \eta_x+\xi_y \eta_y=\xi_r \eta_r\left(\tilde{\xi}_x \tilde{\eta}_x+\tilde{\xi}_y \tilde{\eta}_y\right)=\xi_r \eta_r\left(-\cos\theta\sin\theta+\cos\theta\sin\theta\right)=0$$

$$\eta_x{}^2+\eta_y{}^2=\eta_r{}^2\left(\tilde{\eta}_x{}^2+\tilde{\eta}_y{}^2\right)=\eta_r{}^2\left(\sin^2\theta+\cos^2\theta\right)=\eta_r{}^2$$

$$J=\xi_x \eta_y-\xi_y \eta_x=\xi_r \eta_r\left(\tilde{\xi}_x \tilde{\eta}_y-\tilde{\xi}_y \tilde{\eta}_x\right)=\xi_r \eta_r\left(\sin^2\theta+\cos^2\theta\right)=\xi_r \eta_r$$

（

一般座標上の格子サイズ　$\Delta\xi, \Delta\eta$

局所的な格子サイズ　$\Delta\tilde{\xi}, \Delta\tilde{\eta}$

これらの比を　$\xi_r, \eta_r = \dfrac{\Delta\xi}{\Delta\tilde{\xi}}, \dfrac{\Delta\eta}{\Delta\tilde{\eta}}$

よって$\xi_x, \xi_y, \eta_x, \eta_y$は

$\xi_x=\xi_r\tilde{\xi}_x$　,　$\xi_y=\xi_r\tilde{\xi}_y$　,　$\eta_x=\eta_r\tilde{\eta}_x$　,　$\eta_y=\eta_r\tilde{\eta}_y$

）

――　以上をまとめると、ξ方向は

$$D^\xi=\frac{\partial}{\partial \xi}\left(\nu_T \xi_r{}^2 \frac{\partial u^\xi}{\partial \xi}\right)+\frac{\partial}{\partial \eta}\left(\nu_T \eta_r{}^2 \frac{\partial u^\xi}{\partial \eta}\right)$$

同様にη方向も展開すると

$$D^{\eta}=\frac{\partial}{\partial\xi}\left(\nu_T{\xi_r}^2\frac{\partial u^{\eta}}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\nu_T{\eta_r}^2\frac{\partial u^{\eta}}{\partial\eta}\right)$$

——　浮遊砂の拡散についても同様に展開すると

$$D^{\xi}=\frac{\partial}{\partial\xi}\left(\nu_T{\xi_r}^2\frac{\partial c}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(\nu_T{\eta_r}^2\frac{\partial c}{\partial\eta}\right)$$

間違い、不明点等ありましたら、ご連絡ください。

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