In this paper the attempt which 3-d continuty equations are transformed to non-orthogonal system is made.

3-dimensional continuty and momentum equations in co-orthogonal coordinate are written as follow.

Basic equations in orthogonal coordinate

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] - g \tag{4}$$

in which x,y and z are co-orthogonal coordinates and u,v and w = flow velocity in x,y and z directions, respectively. rho is the fluid density, g is acceleration of gravity, and nu is kinematic viscosity.

$$\frac{\partial}{\partial t} = \tau_t\frac{\partial}{\partial \tau} + \xi_t\frac{\partial}{\partial \xi} + \eta_t\frac{\partial}{\partial \eta} + \sigma_t\frac{\partial}{\partial \sigma} \tag{5}$$

$$\frac{\partial}{\partial x} = \tau_x\frac{\partial}{\partial \tau} + \xi_x\frac{\partial}{\partial \xi} + \eta_x\frac{\partial}{\partial \eta} + \sigma_x\frac{\partial}{\partial \sigma} \tag{6}$$

$$\frac{\partial}{\partial y} = \tau_y\frac{\partial}{\partial \tau} + \xi_y\frac{\partial}{\partial \xi} + \eta_y\frac{\partial}{\partial \eta} + \sigma_y\frac{\partial}{\partial \sigma} \tag{7}$$

$$\frac{\partial}{\partial z} = \tau_z\frac{\partial}{\partial \tau} + \xi_z\frac{\partial}{\partial \xi} + \eta_z\frac{\partial}{\partial \eta} + \sigma_z\frac{\partial}{\partial \sigma} \tag{8}$$

Deformation Procedure

$$\begin{aligned}
\begin{pmatrix} \tau_t & \xi_t & \eta_t & \sigma_t \\ \tau_x & \xi_x & \eta_x & \sigma_x \\ \tau_y & \xi_y & \eta_y & \sigma_y \\ \tau_z & \xi_z & \eta_z & \sigma_z \end{pmatrix}
&= \begin{pmatrix} t_\tau & x_\tau & y_\tau & z_\tau \\ t_\xi & x_\xi & y_\xi & z_\xi \\ t_\eta & x_\eta & y_\eta & z_\eta \\ t_\sigma & x_\sigma & y_\sigma & z_\sigma \end{pmatrix}^{-1} \\
&= \begin{pmatrix} 1 & x_\tau & y_\tau & z_\tau \\ 0 & x_\xi & y_\xi & z_\xi \\ 0 & x_\eta & y_\eta & z_\eta \\ 0 & 0 & 0 & z_\sigma \end{pmatrix}^{-1} \\
&= \frac{1}{J}\begin{pmatrix} J & -x_\tau y_\eta z_\sigma + x_\eta y_\tau z_\sigma & x_\tau y_\xi z_\sigma - x_\xi y_\tau z_\sigma & \beta \\ 0 & y_\eta z_\sigma & -y_\xi z_\sigma & y_\xi z_\eta - y_\eta z_\xi \\ 0 & -x_\eta z_\sigma & x_\xi z_\sigma & -x_\xi z_\eta + x_\eta z_\xi \\ 0 & 0 & 0 & x_\xi y_\eta - x_\eta y_\xi \end{pmatrix} \\
&= \begin{pmatrix} 1 & (-x_\tau y_\eta + x_\eta y_\tau) z_\sigma & (x_\tau y_\xi - x_\xi y_\tau) z_\sigma & \beta / J \\ 0 & y_\eta / J' & -y_\xi / J' & (y_\xi z_\eta - y_\eta z_\xi)/J \\ 0 & -x_\eta / J' & x_\eta / J' & (-x_\xi z_\eta + x_\eta z_\xi)/J \\ 0 & 0 & 0 & 1/z_\sigma \end{pmatrix}
\end{aligned} \tag{9}$$

where,

$$J = x_\xi y_\eta z_\sigma - x_\eta y_\xi z_\sigma$$

$$\beta = -x_\tau y_\xi z_\eta + x_\xi y_\tau z_\eta + x_\tau y_\eta z_\xi - x_\eta y_\tau z_\xi - x_\xi y_\eta z_\tau + x_\eta y_\xi z_\tau$$

Deformation of velocity

$$U = \xi_x u + \xi_y v$$
$$V = \eta_x u + \eta_y v$$
$$W = \sigma_x u + \sigma_y v + \sigma_z w$$

Continuty equation

$$\frac{\partial (JU)}{\partial \xi} + \frac{\partial (JV)}{\partial \eta} + \frac{\partial (JW)}{\partial \sigma} = 0 \tag{10}$$

Momentum equation in ξ-direciton

$$\begin{aligned}
&\frac{\partial U}{\partial \tau} + (\xi_t + U)\frac{\partial U}{\partial \xi} + (\eta_t + V)\frac{\partial U}{\partial \eta} + (\sigma_t + W)\frac{\partial U}{\partial \sigma} + \alpha_1 U + \alpha_2 V + \xi_t(\alpha_3 U + \alpha_4 V) \\
&+ \eta_t(\alpha_4 U + \alpha_5 V) + \alpha_3 U^2 + \alpha_5 V^2 + 2\alpha_4 UV \\
= &-\frac{1}{\rho}\left\{\left({\xi_x}^2 + {\xi_y}^2\right)\frac{\partial p}{\partial \xi} + (\xi_x\eta_x + \xi_y\eta_y)\frac{\partial p}{\partial \eta} + (\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial p}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \xi}\left\{\nu\left({\xi_x}^2 + {\xi_y}^2\right)\frac{\partial U}{\partial \xi}\right\} + \frac{\partial}{\partial \xi}\left\{\nu(\xi_x\eta_x + \xi_y\eta_y)\frac{\partial U}{\partial \eta}\right\} + \frac{\partial}{\partial \xi}\left\{\nu(\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial U}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \eta}\left\{\nu(\xi_x\eta_x + \xi_y\eta_y)\frac{\partial U}{\partial \xi}\right\} + \frac{\partial}{\partial \eta}\left\{\nu\left({\eta_x}^2 + {\eta_y}^2\right)\frac{\partial U}{\partial \eta}\right\} + \frac{\partial}{\partial \eta}\left\{\nu(\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial U}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \sigma}\left\{\nu(\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial U}{\partial \xi}\right\} + \frac{\partial}{\partial \sigma}\left\{\nu(\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial U}{\partial \eta}\right\} + \frac{\partial}{\partial \sigma}\left\{\nu\left({\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2\right)\frac{\partial U}{\partial \sigma}\right\}
\end{aligned} \tag{11}$$

where,

$$\alpha_1 = \xi_x\frac{\partial^2 x}{\partial\tau\partial\xi} + \xi_y\frac{\partial^2 y}{\partial\tau\partial\xi} \quad \alpha_2 = \xi_x\frac{\partial^2 x}{\partial\tau\partial\eta} + \xi_y\frac{\partial^2 y}{\partial\tau\partial\eta}$$

$$\alpha_3 = \xi_x\frac{\partial^2 x}{\partial\xi^2} + \xi_y\frac{\partial^2 y}{\partial\xi^2} \qquad \alpha_4 = \xi_x\frac{\partial^2 x}{\partial\xi\partial\eta} + \xi_y\frac{\partial^2 y}{\partial\xi\partial\eta} \quad \alpha_5 = \xi_x\frac{\partial^2 x}{\partial\eta^2} + \xi_y\frac{\partial^2 y}{\partial\eta^2}$$

Momentum equation in η-direction

$$
\begin{aligned}
&\frac{\partial V}{\partial \tau} + (\xi_t + U)\frac{\partial V}{\partial \xi} + (\eta_t + V)\frac{\partial V}{\partial \eta} + (\sigma_t + W)\frac{\partial V}{\partial \sigma} + \beta_1 U + \beta_2 V + \xi_t(\beta_3 U + \beta_4 V) \\
&+ \eta_t(\beta_4 U + \beta_5 V) + \beta_3 U^2 + \beta_5 V^2 + 2\beta_4 UV \\
= &-\frac{1}{\rho}\left\{(\xi_x\eta_x + \xi_y\eta_y)\frac{\partial p}{\partial \xi} + \left({\eta_x}^2 + {\eta_y}^2\right)\frac{\partial p}{\partial \eta} + (\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial p}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \xi}\left\{\nu\left({\xi_x}^2 + {\xi_y}^2\right)\frac{\partial V}{\partial \xi}\right\} + \frac{\partial}{\partial \xi}\left\{\nu(\xi_x\eta_x + \xi_y\eta_y)\frac{\partial V}{\partial \eta}\right\} + \frac{\partial}{\partial \xi}\left\{\nu(\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial V}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \eta}\left\{\nu(\xi_x\eta_x + \xi_y\eta_y)\frac{\partial V}{\partial \xi}\right\} + \frac{\partial}{\partial \eta}\left\{\nu\left({\eta_x}^2 + {\eta_y}^2\right)\frac{\partial V}{\partial \eta}\right\} + \frac{\partial}{\partial \eta}\left\{\nu(\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial V}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \sigma}\left\{\nu(\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial V}{\partial \xi}\right\} + \frac{\partial}{\partial \sigma}\left\{\nu(\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial V}{\partial \eta}\right\} + \frac{\partial}{\partial \sigma}\left\{\nu\left({\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2\right)\frac{\partial V}{\partial \sigma}\right\}
\end{aligned} \tag{12}
$$

where,

$$
\beta_1 = \eta_x\frac{\partial^2 x}{\partial \tau \partial \xi} + \eta_y\frac{\partial^2 y}{\partial \tau \partial \xi} \quad \beta_2 = \eta_x\frac{\partial^2 x}{\partial \tau \partial \eta} + \eta_y\frac{\partial^2 y}{\partial \tau \partial \eta}
$$

$$
\beta_3 = \eta_x\frac{\partial^2 x}{\partial \xi^2} + \eta_y\frac{\partial^2 y}{\partial \xi^2} \quad \beta_4 = \eta_x\frac{\partial^2 x}{\partial \xi \partial \eta} + \eta_y\frac{\partial^2 y}{\partial \xi \partial \eta} \quad \beta_5 = \eta_x\frac{\partial^2 x}{\partial \eta^2} + \eta_y\frac{\partial^2 y}{\partial \eta^2}
$$

Momentum equation σ-direction

$$
\begin{aligned}
&\frac{\partial W}{\partial \tau} + (U + \xi_t)\frac{\partial W}{\partial \xi} + (V + \eta_t)\frac{\partial W}{\partial \eta} + (W + \sigma_t)\frac{\partial W}{\partial \sigma} + \gamma_1 U + \gamma_2 V + \gamma_3 W \\
&+ \xi_t(\gamma_4 U + \gamma_5 V + \gamma_6 W) + \eta_t(\gamma_5 U + \gamma_7 V + \gamma_8 W) + \sigma_t(\gamma_6 + \gamma_8 + \gamma_9) \\
&+ \gamma_4 U^2 + \gamma_7 V^2 + \gamma_9 W^2 + 2\gamma_5 UV + 2\gamma_6 UW + 2\gamma_8 VW \\
= &-\frac{1}{\rho}\left\{(\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial p}{\partial \xi} + (\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial p}{\partial \eta} + \left({\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2\right)\frac{\partial p}{\partial \sigma}\right\} \\
&+ \frac{\partial}{\partial \xi}\left[\nu\left\{\left({\xi_x}^2 + {\xi_y}^2\right)\frac{\partial W}{\partial \xi} + (\xi_x\eta_x + \xi_y\eta_y)\frac{\partial W}{\partial \eta} + (\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial W}{\partial \sigma}\right\}\right] \\
&+ \frac{\partial}{\partial \eta}\left[\nu\left\{(\xi_x\eta_x + \xi_y\eta_y)\frac{\partial W}{\partial \xi} + \left({\eta_x}^2 + {\eta_y}^2\right)\frac{\partial W}{\partial \eta} + (\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial W}{\partial \sigma}\right\}\right] \\
&+ \frac{\partial}{\partial \sigma}\left[\nu\left\{(\xi_x\sigma_x + \xi_y\sigma_y)\frac{\partial W}{\partial \xi} + (\eta_x\sigma_x + \eta_y\sigma_y)\frac{\partial W}{\partial \eta} + \left({\sigma_x}^2 + {\sigma_y}^2 + {\sigma_z}^2\right)\frac{\partial W}{\partial \sigma}\right\}\right] - \sigma_z g
\end{aligned} \tag{13}
$$

where,

$$
\begin{aligned}
\gamma_1 &= \sigma_x\frac{\partial^2 x}{\partial \tau \partial \xi} + \sigma_y\frac{\partial^2 y}{\partial \tau \partial \xi} + \sigma_z\frac{\partial^2 z}{\partial \tau \partial \xi} & \gamma_2 &= \sigma_x\frac{\partial^2 x}{\partial \tau \partial \eta} + \sigma_y\frac{\partial^2 y}{\partial \tau \partial \eta} + \sigma_z\frac{\partial^2 z}{\partial \tau \partial \eta} & \gamma_3 &= \sigma_z\frac{\partial^2 z}{\partial \tau \partial \sigma} \\
\gamma_4 &= \sigma_x\frac{\partial^2 x}{\partial \xi^2} + \sigma_y\frac{\partial^2 y}{\partial \xi^2} + \sigma_z\frac{\partial^2 z}{\partial \xi^2} & \gamma_5 &= \sigma_x\frac{\partial^2 x}{\partial \xi \partial \eta} + \sigma_y\frac{\partial^2 y}{\partial \xi \partial \eta} + \sigma_z\frac{\partial^2 z}{\partial \xi \partial \eta} & \gamma_6 &= \sigma_z\frac{\partial^2 z}{\partial \xi \partial \sigma} \\
\gamma_7 &= \sigma_x\frac{\partial^2 x}{\partial \eta^2} + \sigma_y\frac{\partial^2 y}{\partial \eta^2} + \sigma_z\frac{\partial^2 z}{\partial \eta^2} & \gamma_8 &= \sigma_z\frac{\partial^2 z}{\partial \eta \partial \sigma} & \gamma_9 &= \sigma_z\frac{\partial^2 z}{\partial \sigma^2}
\end{aligned}
$$